Mechanics of Thin-walled Structures

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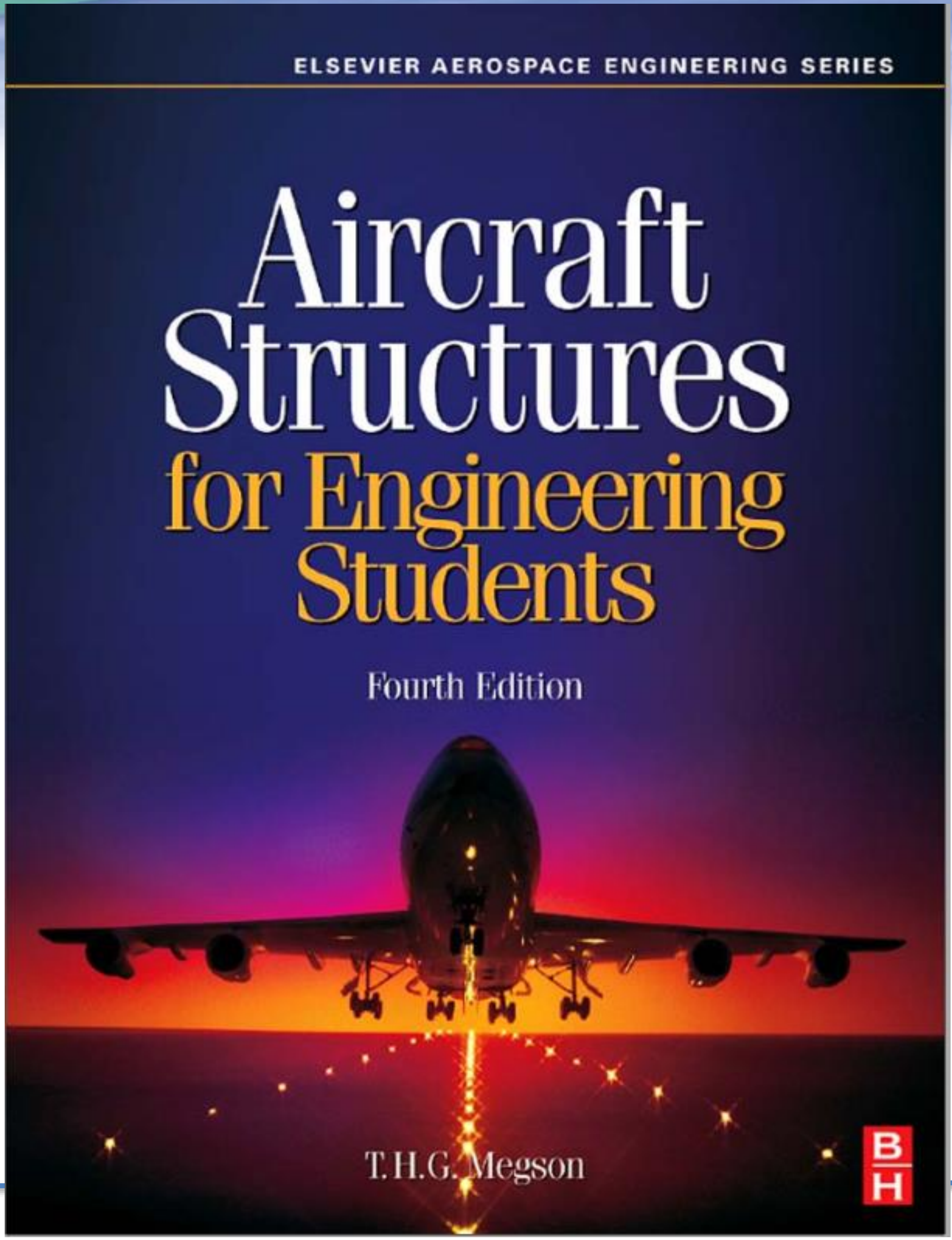
1. Basic concepts of mechanics of structures
    a. Stress
    b. Strain
    c. Moment of inertia
        i. First (static) moment of area (static)
        ii. Second moment of area (inertia)

2. Thin-walled structures introduction

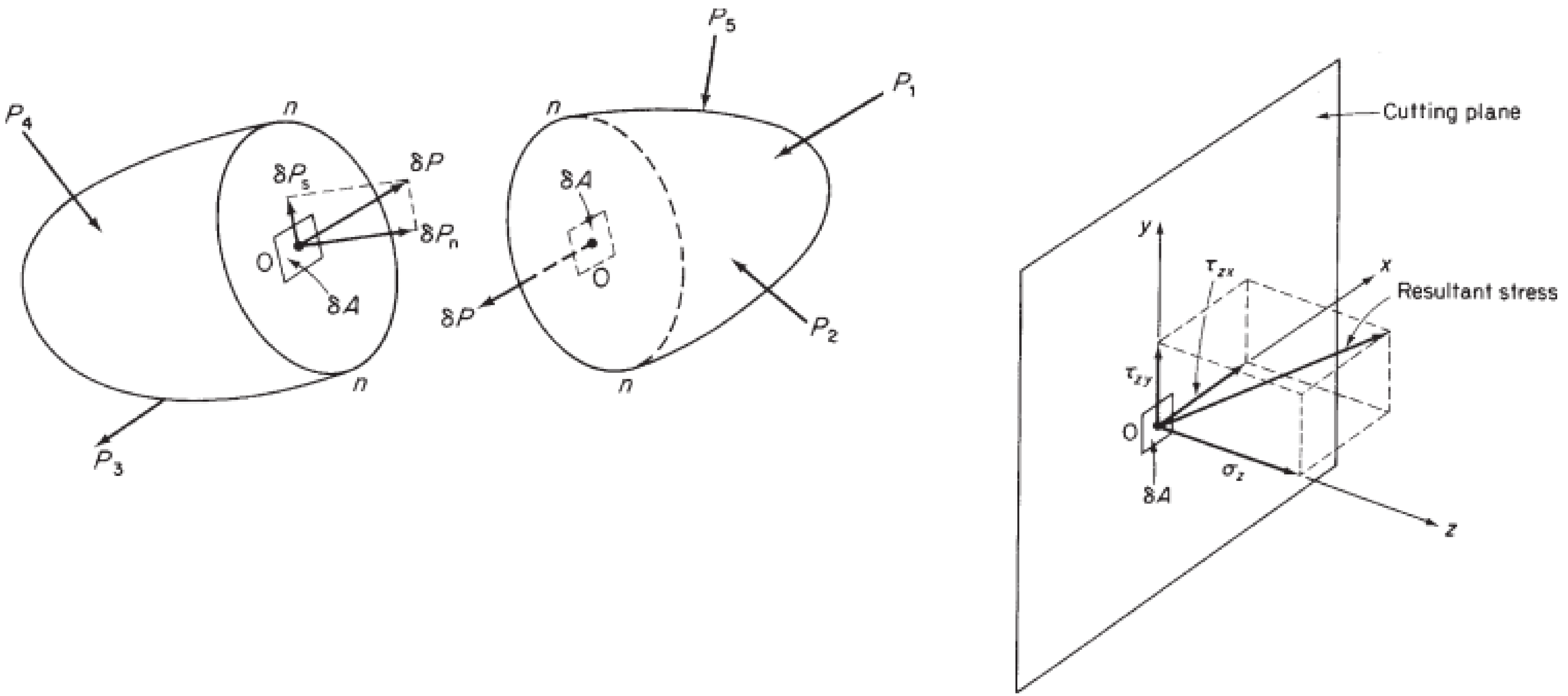
3. Beams
    a. Bending of beams
        i. Shear centre
        ii. Open section beams
            - First (static) moment of inertia approach
            - Function approach
        iii. Closed section beams
    b. Torsion of beams
        i. Free torsion
        ii. Constrained torsion

4. Plates and shells (2D structures)
5. Buckling
   a. Analytical approach
   b. Energy approach
   c. Buckling of columns
   d. Buckling of plates
   e. Buckling of shells


ELSEVIER AEROSPACE ENGINEERING SERIES
Aircraft Structures for Engineering Students
Fourth Edition
T.H.G. Megson
B H

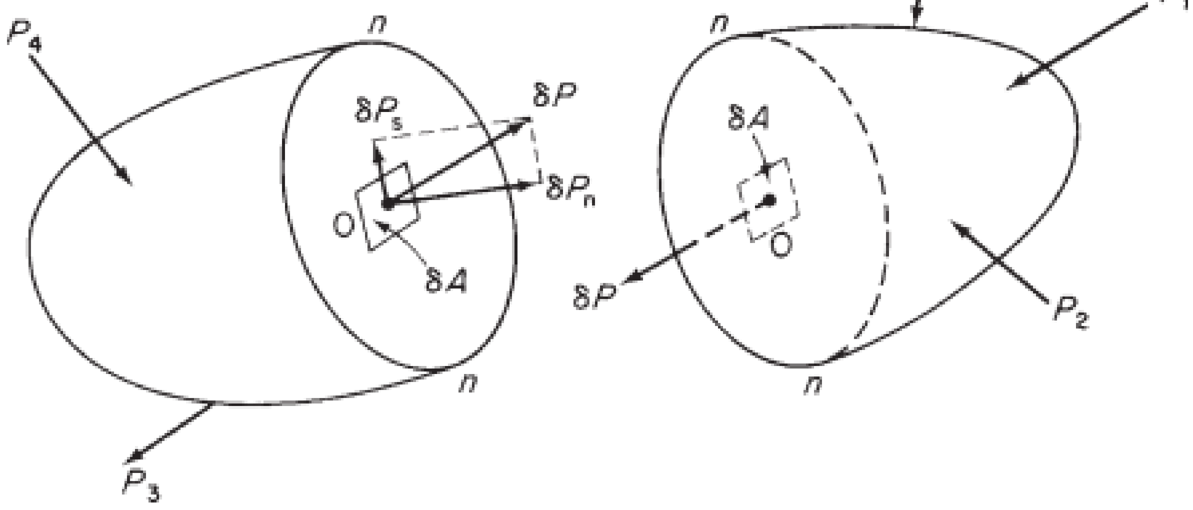
Stress, Strain, Hooke`s law, moments of area (inertia)

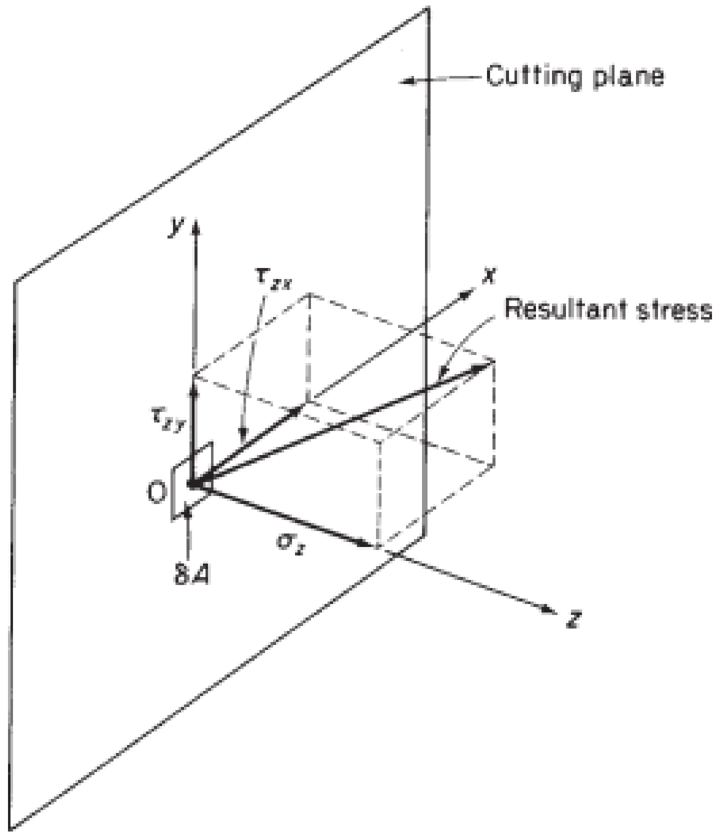
State of stress



State of stress

3D body is in equilibrum under action of externally applied forces P.

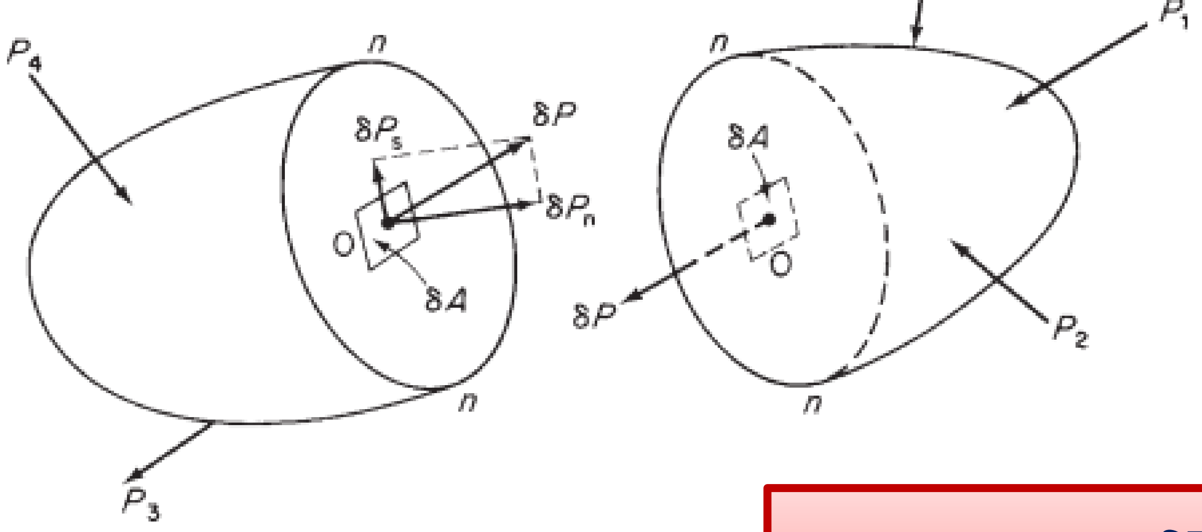


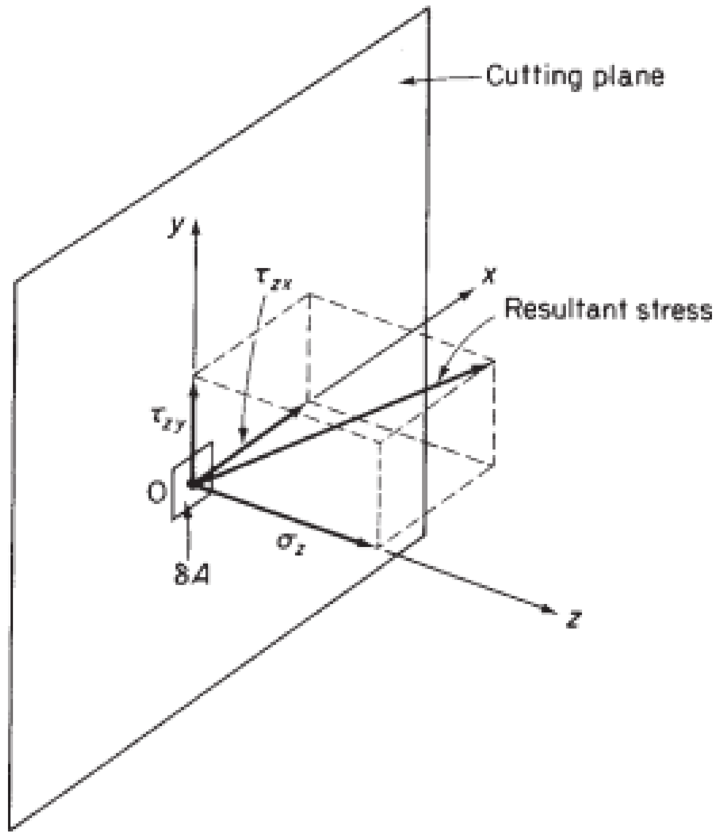


P – forces
O – point in the body
δP – resultant force
nn – plane which divides the body into two parts
δA – small area

State of stress

3D body is in equilibrum under action of externally applied forces P.





P – forces
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$$Stress = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$

State of stress

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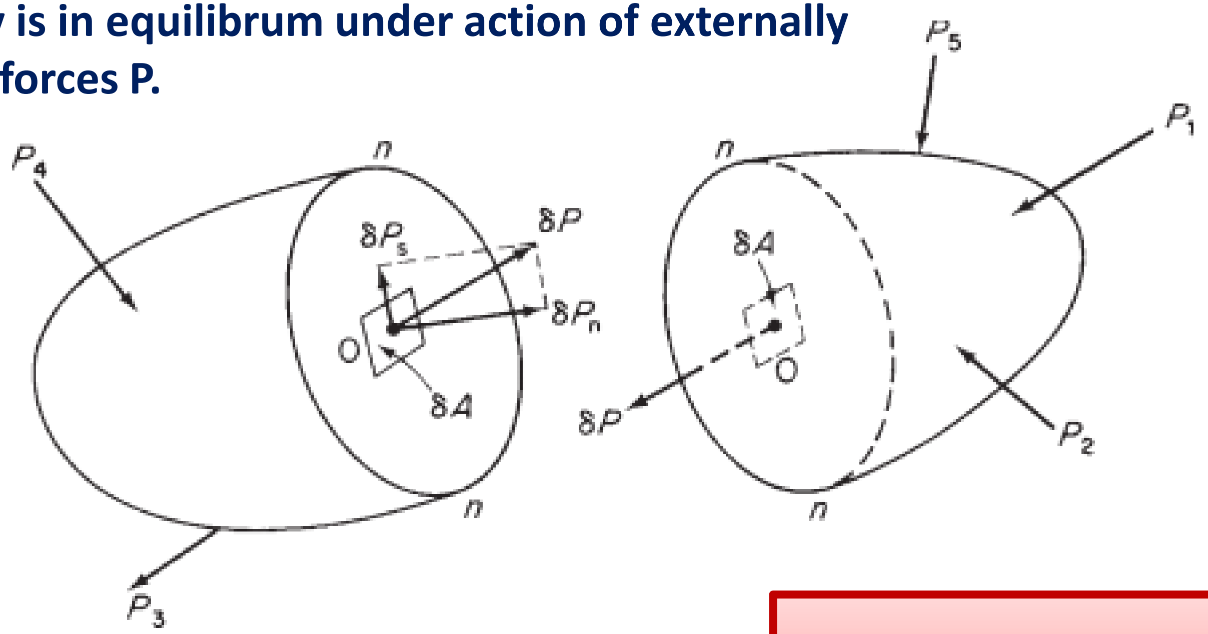


P – forces
O – point in the body
δP – resultant force
nn – plane which divides the body into two parts
δA – small area

$$Stress = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$

The direction of δP is not normal to the area δA, in which case it is usual to resolve δP into two components:
δP_n - normal to the plane
δP_s - acting in the plane itself

Plane conatining δP is perpendicular to δA. The stresses associated with the above-mentioned components are:

Normal stress or direct stress

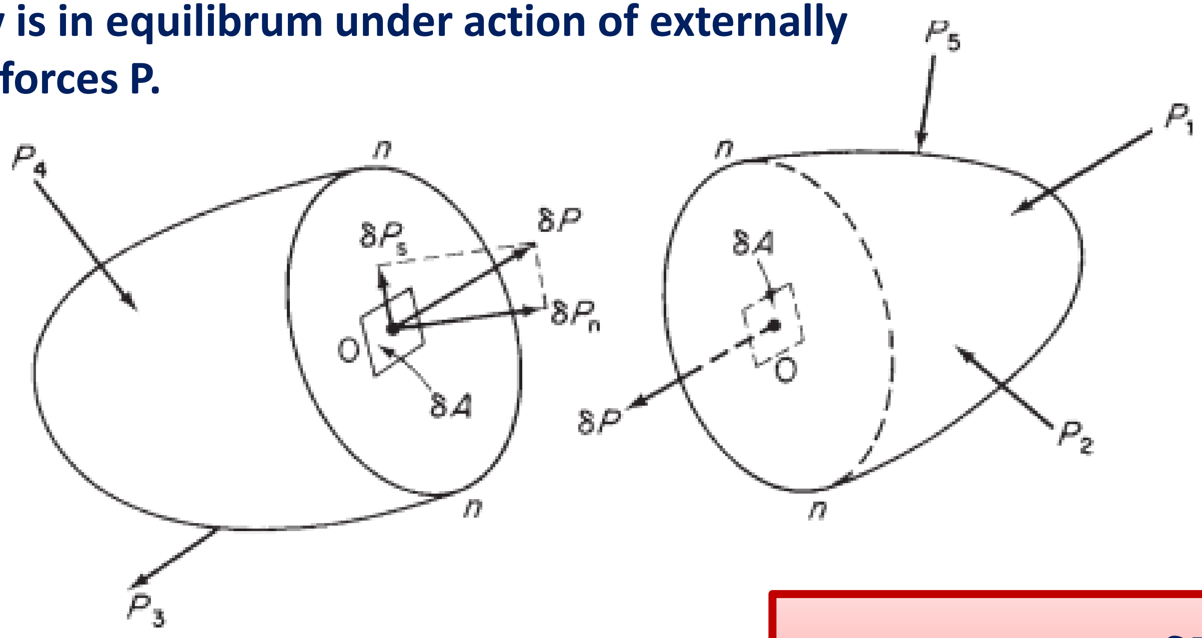
$$\sigma = \lim_{\delta A \to 0} \frac{\delta P_n}{\delta A}$$

Shear stress

$$\tau = \lim_{\delta A \to 0} \frac{\delta P_s}{\delta A}$$

State of stress

3D body is in equilibrum under action of externally applied forces P.



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O – point in the body
δP – resultant force
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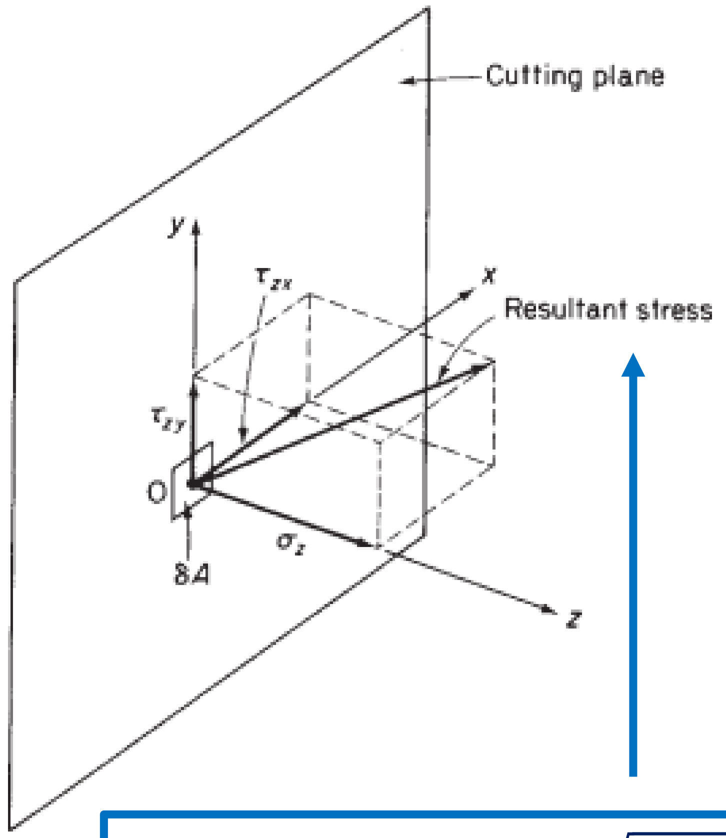
$$Stress = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$



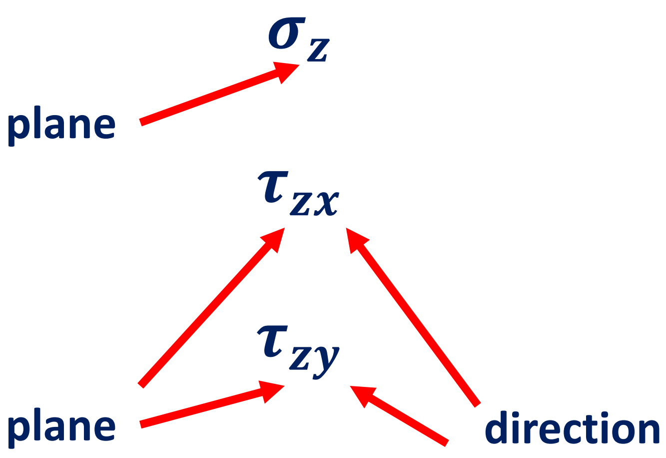
$$Resultant\ stress = \sqrt{\sigma^2 + \tau^2}$$

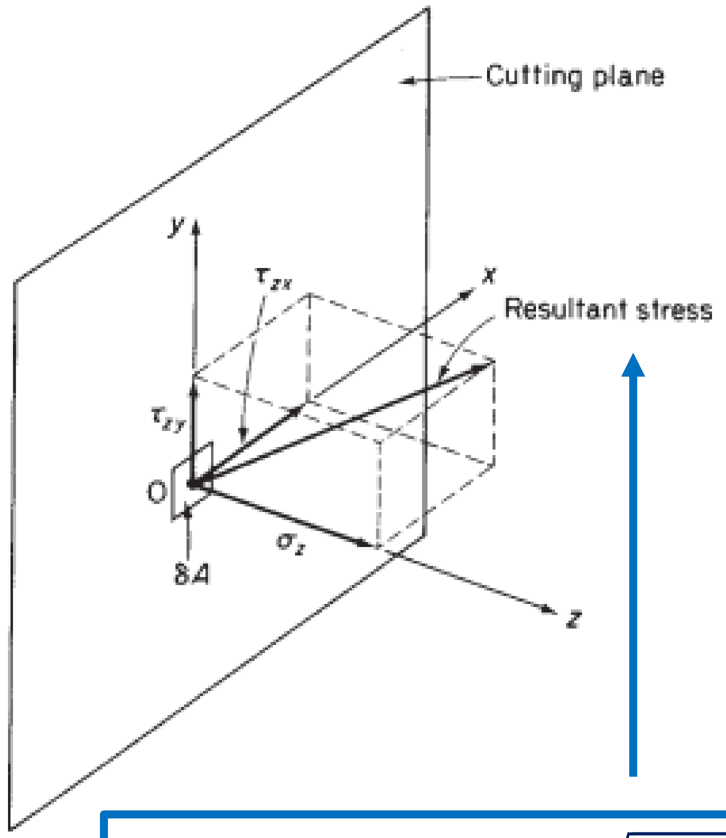
State of stress

The resultant force δP may be resolved into the following components:

- one normal component

- two in-plane components of shear stress





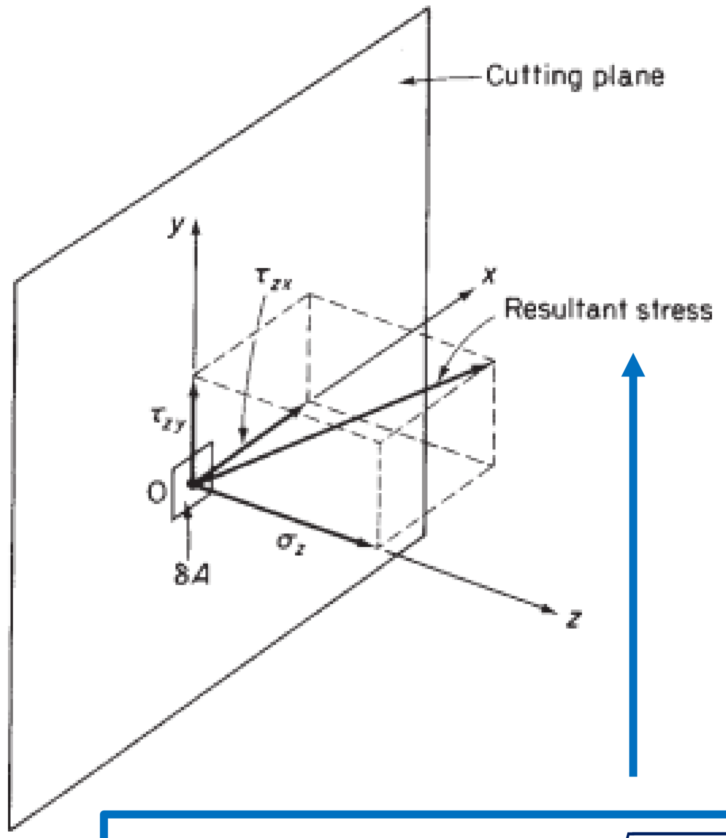
$$Resultant\ stress = \sqrt{\sigma^2 + \tau^2}$$

State of stress

What is the stress?

➔vector

➔tensor



$$Stress = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$

$$Resultant\ stress = \sqrt{\sigma^2 + \tau^2}$$

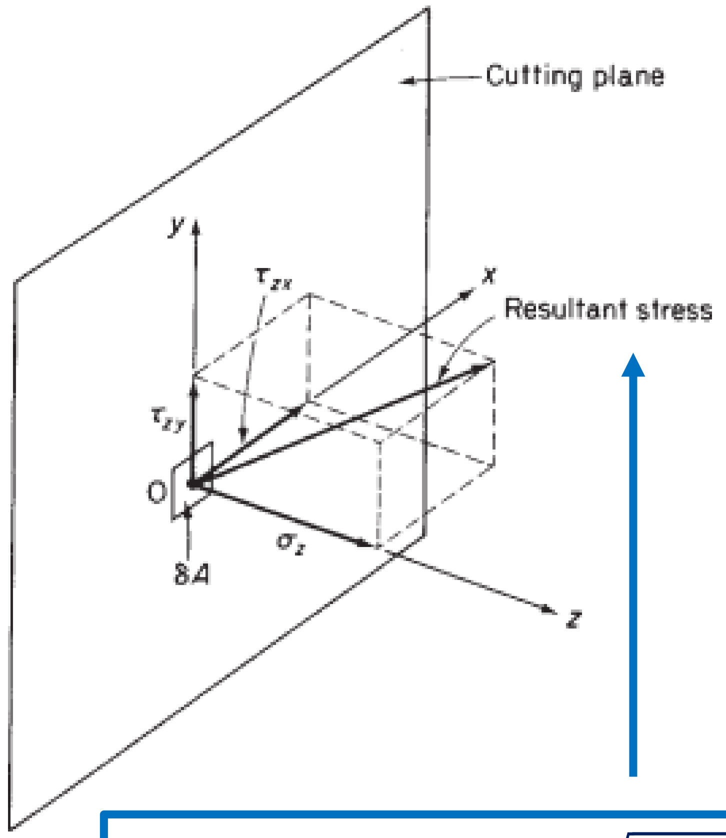
State of stress

What is the stress?
➔vector
➔tensor

The following elements must be specified to determine the stress:

- **magnitude**
- **direction**
- **plane on which stress acts**

$$Stress = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$



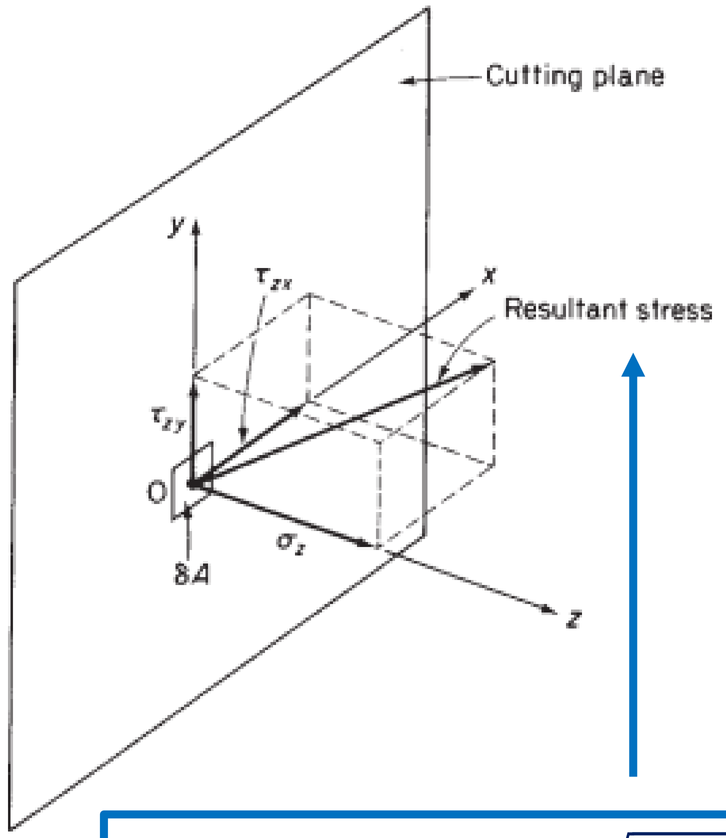
$$Resultant\ stress = \sqrt{\sigma^2 + \tau^2}$$

State of stress

What is the unit of stress?

$$\left[\frac{N}{m^2}\right] = [Pa]$$

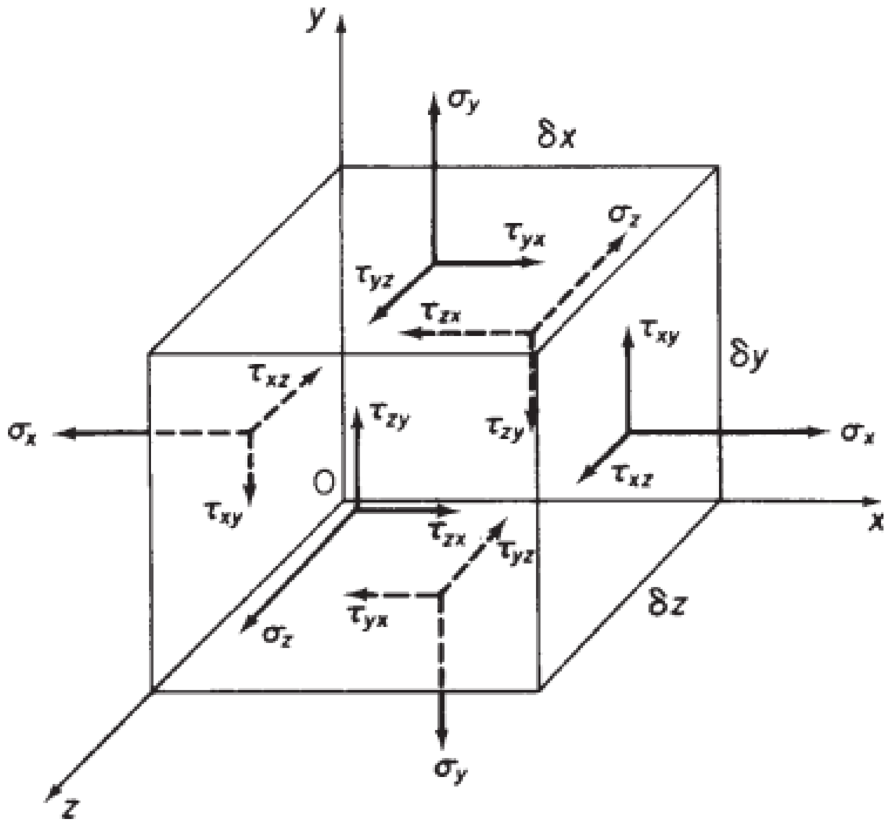
$$Stress = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$

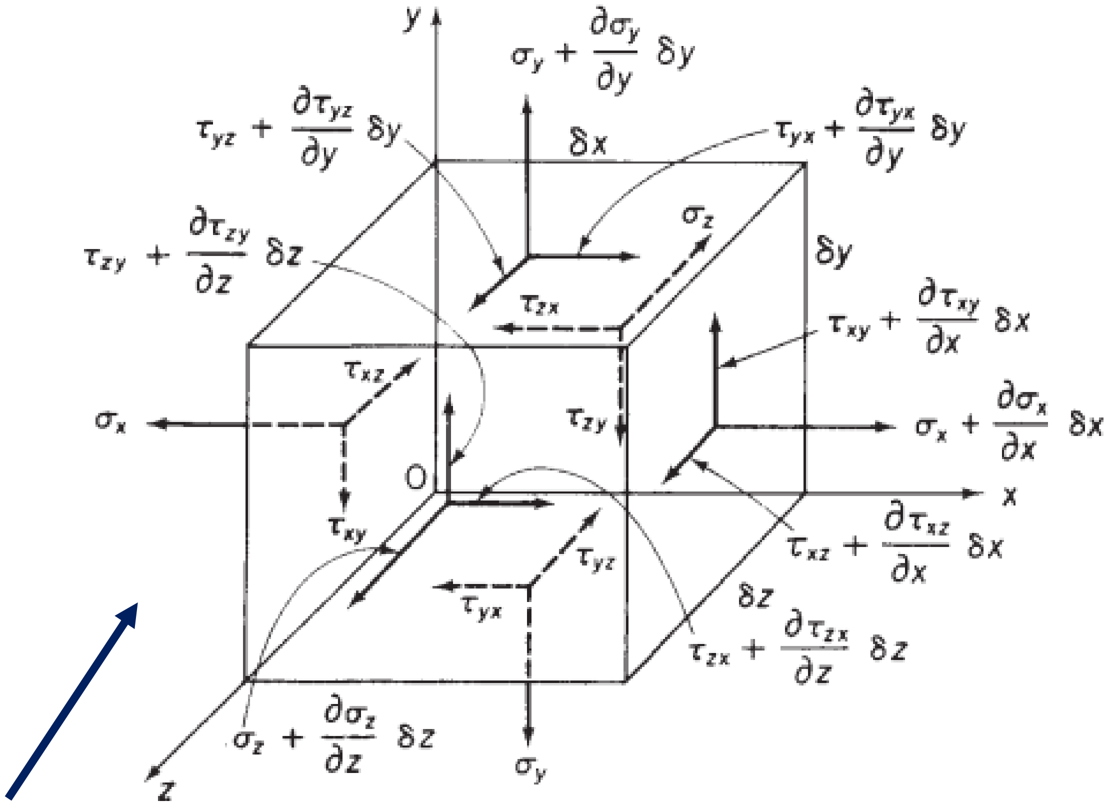


$$Resultant\ stress = \sqrt{\sigma^2 + \tau^2}$$

State of stress

Normal and shear stresses:





Generally, except in cases of uniform stress, the normal and shear stresses on opposite faces of an element are not equal but differ by small amounts.

State of stress

After solving, the equations of equilibrum can be written:

$$\left.\begin{aligned}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0 \\
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0 \\
\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + Z = 0
\end{aligned}\right\}$$

X, Y, Z – body forces coming from gravitational forces and inertia effects

State of stress

After solving, the equations of equilibrum can be written:

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0 \\ \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + Z = 0 \end{aligned}\right\}$$

X, Y, Z – body forces coming from gravitational forces and inertia effects

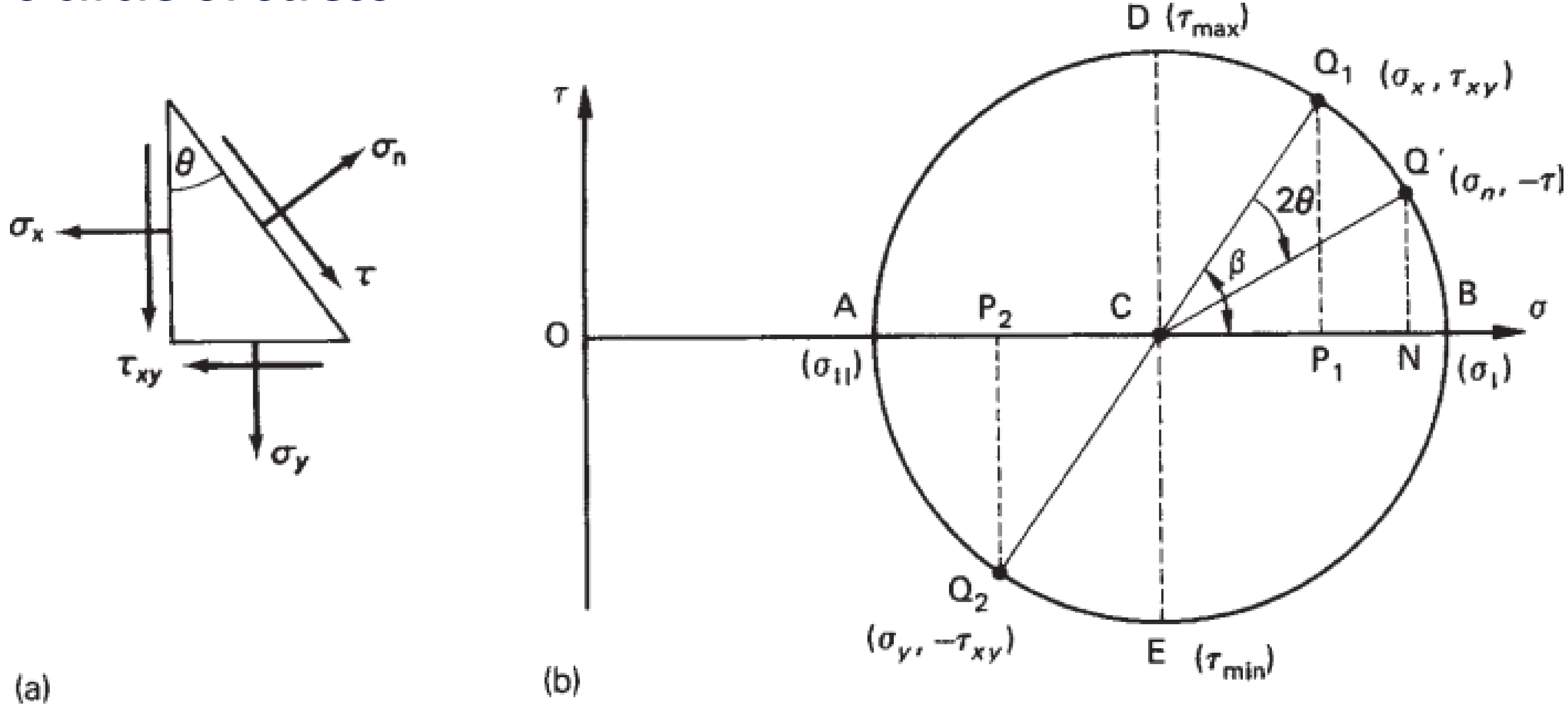
2D case – plane stress:

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + Y = 0 \end{aligned}\right\}$$

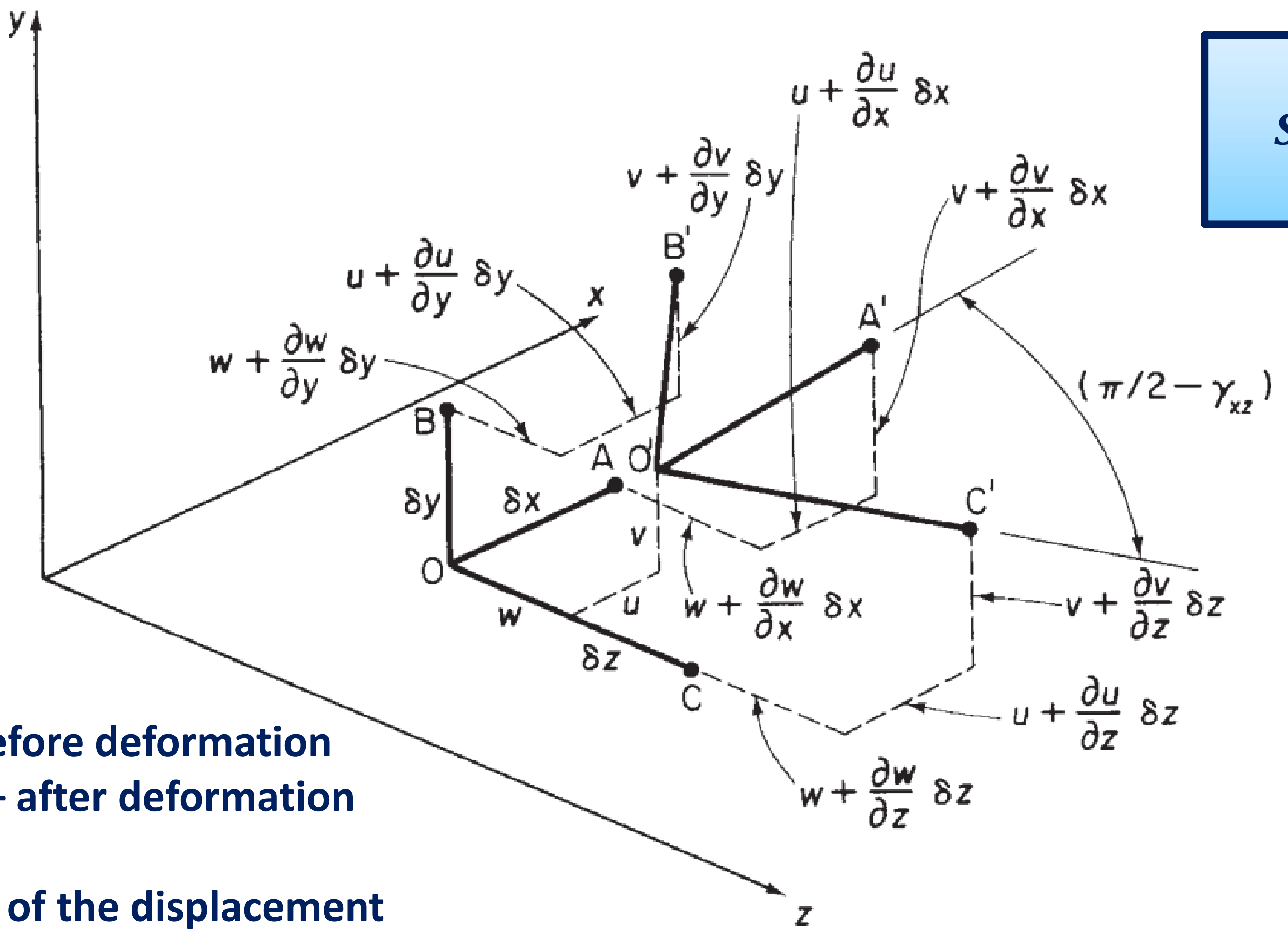
$$\sigma_z = 0$$
$$\tau_{xz} = 0$$
$$\tau_{yz} = 0$$

State of stress

Mohr's circle of stress



State of strain



$$Strain = \lim_{L \to 0} \frac{\Delta L}{L}$$

Lines OA, OB, OC – before deformation
Lines O’A’, O’B’, O’C’ – after deformation

u, v, w – components of the displacement

State of strain

Longitudinal or direct strains:

$$\varepsilon = \lim_{L \to 0} \frac{\Delta L}{L}$$

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ \varepsilon_z &= \frac{\partial w}{\partial z} \end{aligned}\right\}$$

Shear strains:

$$\left.\begin{aligned} \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \end{aligned}\right\}$$

Stress – strain relations
1-D case

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} \qquad \varepsilon_z = -\nu\frac{\sigma_x}{E}$$

Stress – strain relations
3-D case

$$\left.\begin{aligned}\varepsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]\end{aligned}\right\}$$

$$\sigma_x = \frac{\nu E}{(1+\nu)(1-2\nu)}e + \frac{E}{(1+\nu)}\varepsilon_x$$

$$\sigma_y = \frac{\nu E}{(1+\nu)(1-2\nu)}e + \frac{E}{(1+\nu)}\varepsilon_y$$

$$\sigma_z = \frac{\nu E}{(1+\nu)(1-2\nu)}e + \frac{E}{(1+\nu)}\varepsilon_z$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Stress – strain relations
2-D case

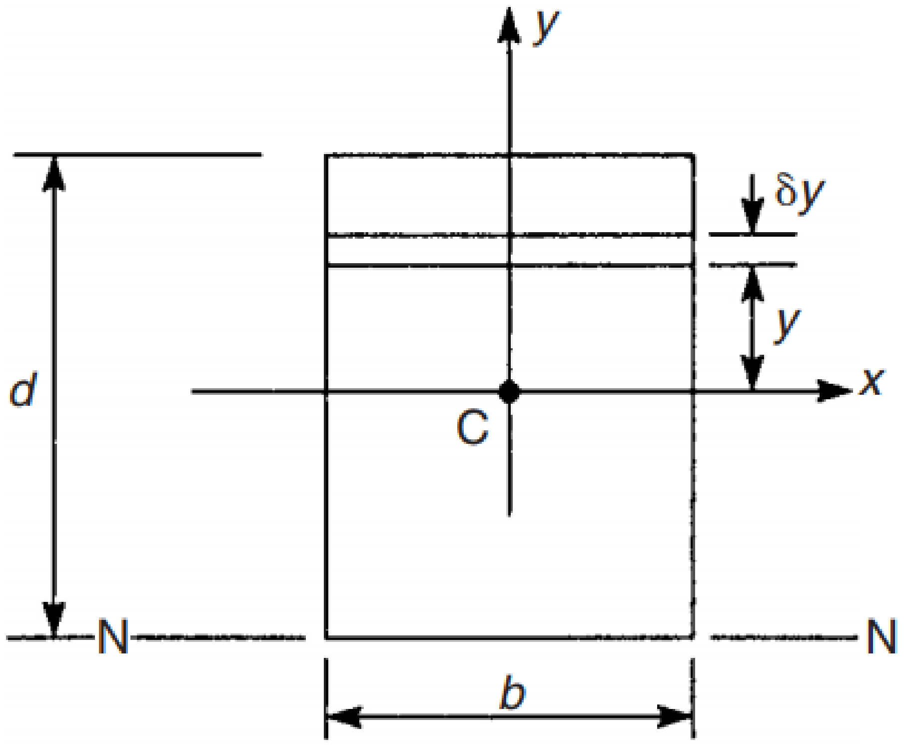
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\gamma = \tau/G$$

$$\gamma = \frac{2(1+\nu)}{E}\tau$$

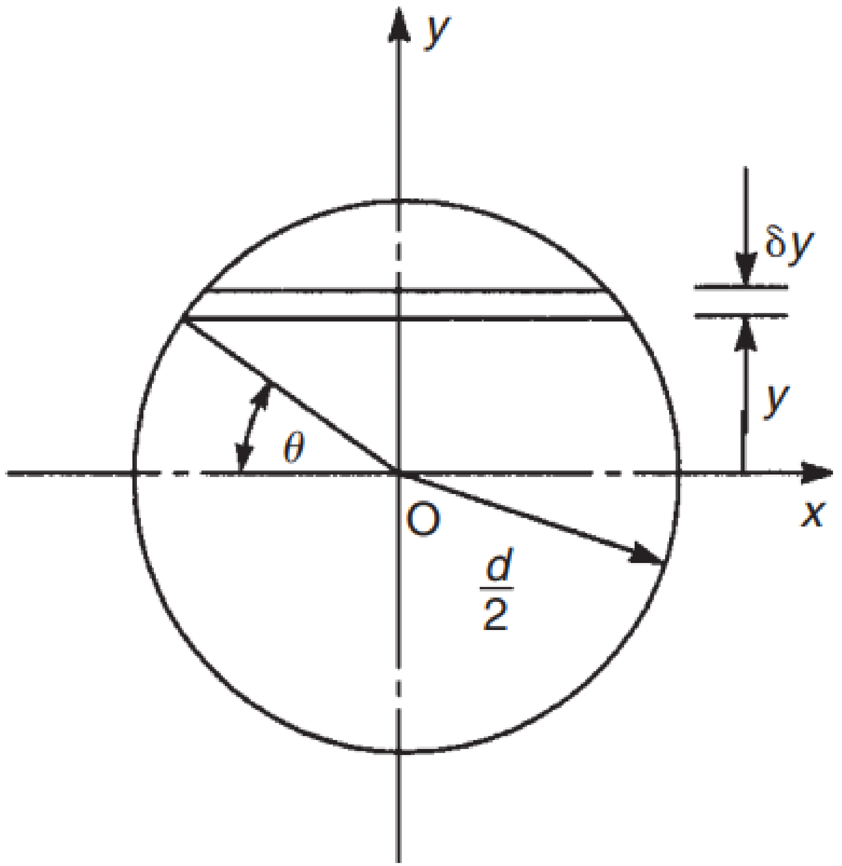
Second moment of area (inertia)



$$I_{xx} = \int_A y^2 \mathrm{d}A = \int_{-d/2}^{d/2} b y^2 \mathrm{d}y = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$I_{xx} = \frac{bd^3}{12}$$

Second moment of area (inertia)



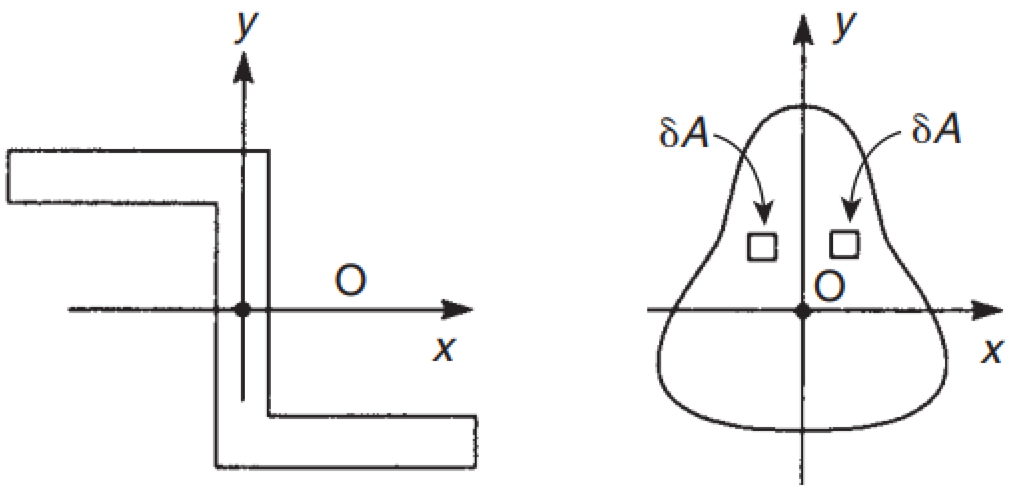
$$I_{xx} = \int_A y^2 \mathrm{d}A = \int_{-d/2}^{d/2} 2\left(\frac{d}{2}\cos\theta\right) y^2 \mathrm{d}y$$

$$I_{xx} = \int_{-\pi/2}^{\pi/2} d\cos\theta \left(\frac{d}{2}\sin\theta\right)^2 \frac{d}{2}\cos\theta \, \mathrm{d}\theta$$

$$I_{xx} = \frac{d^4}{8}\int_{-\pi/2}^{\pi/2} \cos^2\theta \, \sin^2\theta \, \mathrm{d}\theta$$

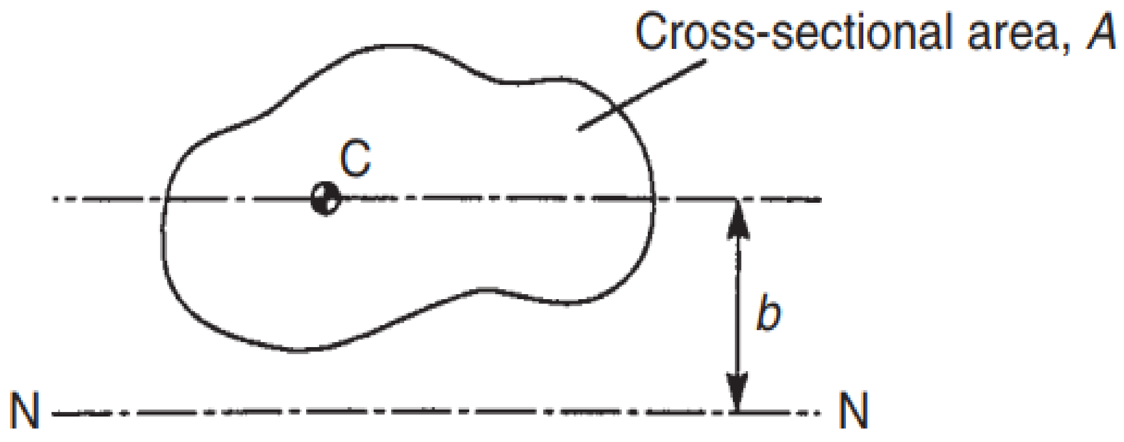
$$I_{xx} = \frac{\pi d^4}{64}$$

Product second moment of area (inertia)



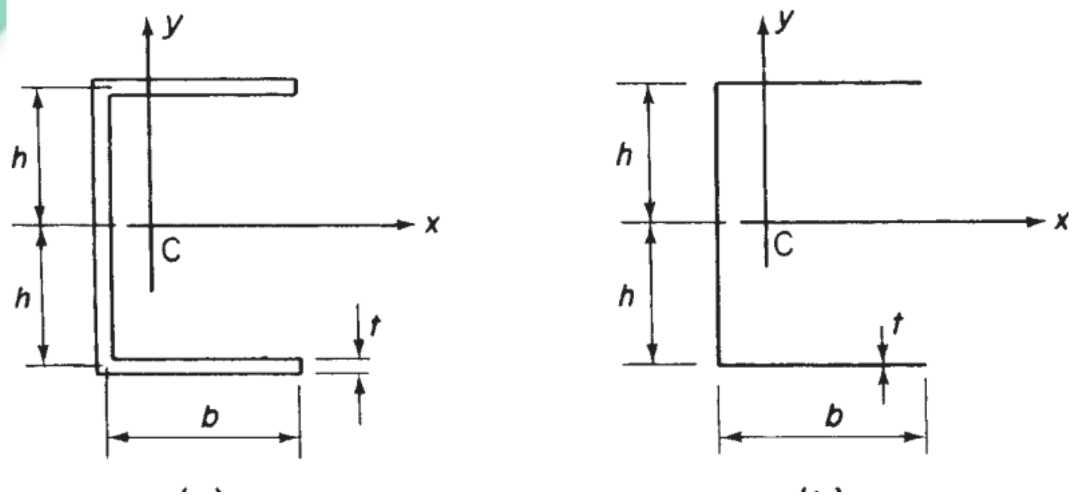
$$I_{xy} = \int_A xy \, \mathrm{d}A$$

Parallel axes theorem (Steiner principle)



$$I_N = I_C + Ab^2$$

Aproximations for Thin-Walled sections

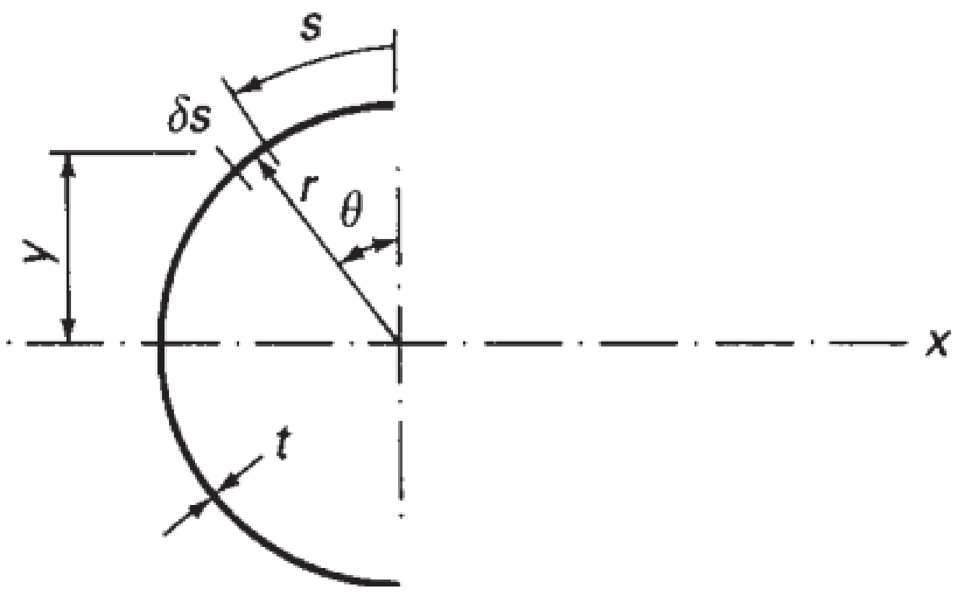


$$I_{xx} = 2\left[\frac{(b+t/2)t^3}{12} + \left(b+\frac{t}{2}\right)th^2\right] + t\frac{[2(h-t/2)]^3}{12}$$

$$I_{xx} = 2\left[\frac{(b+t/2)t^3}{12} + \left(b+\frac{t}{2}\right)th^2\right] + \frac{t}{12}\left[(2)^3\left(h^3 - 3h^2\frac{t}{2} + 3h\frac{t^2}{4} - \frac{t^3}{8}\right)\right]$$

$$I_{xx} = 2bth^2 + t\frac{(2h)^3}{12}$$

Aproximations for Thin-Walled sections

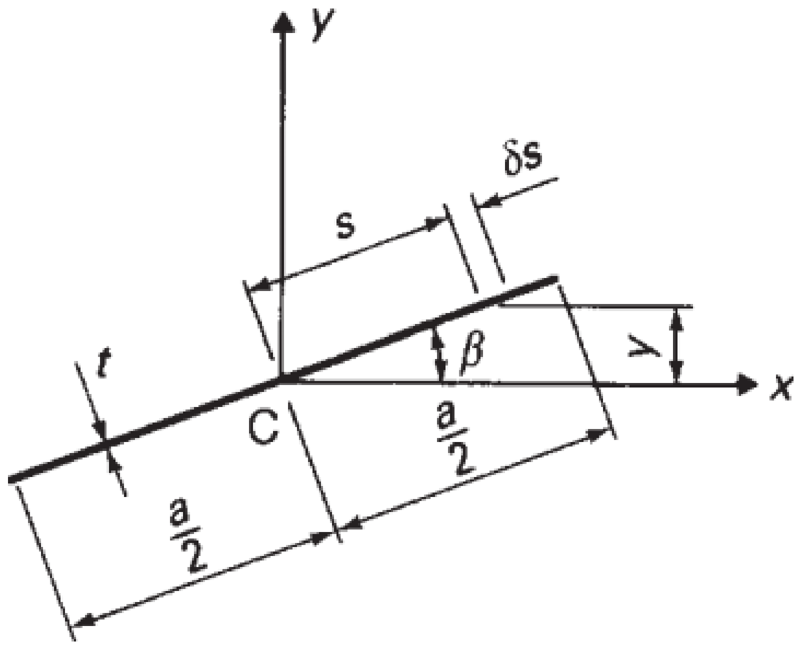


$$I_{xx} = \int_0^{\pi r} t y^2 \, \mathrm{d}s$$

$$I_{xx} = \int_0^{\pi} t (r \cos\theta)^2 r \, \mathrm{d}\theta$$

$$I_{xx} = \frac{\pi r^3 t}{2}$$

Thin-Walled sections – inclined walls



$$I_{xx} = 2\int_0^{a/2} ty^2\,\mathrm{d}s = 2\int_0^{a/2} t(s\sin\beta)^2\,\mathrm{d}s$$

$$I_{xx} = \frac{a^3 t \sin^2\beta}{12}$$

$$I_{yy} = \frac{a^3 t \cos^2\beta}{12}$$

$$I_{xy} = \frac{a^3 t \sin 2\beta}{24}$$